P.G. 1st Semester - 2018

MATHEMATICS

Paper : MMATCCT105

Full Marks : 40 Time : 2 Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meaning.

Answer any **five** questions: 8×5=40

1. What do you mean by action integral? State and verify the principle of least action. 1+7

2. a) Define canonical transformation. Give an example of a phase-space transformation which is not a canonical.

   b) Prove that the transformation:

$$q_1 = \left(\frac{\theta_1}{\omega_1}\right)^{\frac{1}{2}} \cos P_1 + \left(\frac{\theta_2}{\omega_2}\right)^{\frac{1}{2}} \cos P_2$$

$$q_2 = -\left(\frac{\theta_1}{\omega_1}\right)^{\frac{1}{2}} \cos P_1 + \left(\frac{\theta_2}{\omega_2}\right)^{\frac{1}{2}} \cos P_2$$

$$p_1 = (\omega_1\theta_1)^{\frac{1}{2}} \sin P_1 + (\omega_2\theta_2)^{\frac{1}{2}} \sin P_2$$

$$p_2 = -(\omega_1\theta_1)^{\frac{1}{2}} \sin P_1 + (\omega_2\theta_2)^{\frac{1}{2}} \sin P_2$$

is a canonical transformation. If the Hamiltonian of the motion is

$$H = \frac{1}{2}\left(p_1^2 + p_2^2\right) + \frac{1}{4}\omega_1^{\,2}\left(q_1 - q_2\right)^2 + \frac{1}{4}\omega_1^{\,2}\left(q_1 + q_2\right)^2,$$

derive the new Hamiltonian $K(Q_1, Q_2, P_1, P_2)$.

(1+1)+(4+2)

3. a) Show that a time-independent dynamical variable u will be a constant of the motion if the Poisson bracket of u and the corresponding Hamiltonian vanishes.

   b) Using the Hamiltonian function, obtain the equation of motion of a particle of unit mass moving in a central force field, where the force varies inversely as the square of the distance from the centre of force. 4+4

4. a) Obtain the Lagrange's equation of motion of a dynamical system comprising of N number of particles having k holonomic bilateral constraints.

b) Show that all the co-ordinates of a Scleronomous dynamical system are linear function of time, if they are all cyclic.

5+3

5. a) Show that a necessary condition for the functional

$$I\left[y(x)\right]=\int_{x_1}^{x_2} f\left(x,\, y,\, \frac{dy}{dx}\right) dx,$$

define over the space of all functions having continuous first order derivatives on $[x_1, x_2]$ over the real field to be an extremum is that

$$\frac{\partial f}{\partial y}-\frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right)=0,\ \ y'=\frac{dy}{dx},$$

where f is assumed to be twice differentiable with respect to its arguments. Is this condition a sufficient condition? Justify your answer.

b) The Lagrangian L for the motion of a particle of unit mass is given by

$$L=\frac{1}{2}\left(\dot{x}^2+\dot{y}^2+\dot{z}^2\right)-V+\dot{x}A+\dot{y}B+\dot{z}C$$

where V, A, B, C are functions of (x, y, z). Show that the equation of motion are

$$\ddot{x}=-\frac{\partial V}{\partial x}+\dot{y}\left(\frac{\partial B}{\partial x}-\frac{\partial A}{\partial y}\right)-\dot{z}\left(\frac{\partial A}{\partial z}-\frac{\partial C}{\partial x}\right)$$

and two similar equation. 5+3

6. a) Obtain Euler's equations of motion for a rigid body rotating about a point fixed in the body.

b) For a Scleronomous dynamical system of n degrees of freedom prove that the kinetic energy is a homogeneous quadratic function of the generalised velocities. 4+4

7. a) Prove that shortest distance between any two points in a plane is a straight line.

b) Obtain Hamiltons' equation of motion of a dynamical system whose Hamiltonian is given by

$$H=q_1p_1-q_2p_2-aq_1^2+bq_2^2$$

Hence solve the problem. 3+(2+3)

8. a) Establish the Jacobi's Identity for the dynamical variables X, Y and Z in terms of Poisson brackets.

b) If $H=qp^2-pq+bp$, where b is constant, then show that,

$$p=\frac{1}{2}\left(1+\coth\frac{t+\varepsilon}{2}\right),$$

$$q=c\sinh^2\frac{t+\varepsilon}{2}-b\sinh\left(\frac{t+\varepsilon}{2}\right)$$

Also show that $H=\frac{b}{2}+\frac{c}{4}$, where c and ε are arbitrary constant. 4+4

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