P.G. 1st Semester - 2017

MATHEMATICS

Paper : MMATCCT-104

Full Marks : 40 Time : 2 Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and Symbols have their usual meaning.

Answer any **five** questions: $8\times5=40$

1. a) Find the power series solution of the differential equation

$$y''(x)-2xy'(x)+2py(x)=0$$

about $x=0$. Where, p is a constant.

b) Prove that, Legendre polynomial $P_n(z)$ is an even or odd function of z according to n is even or odd. 5+3

2. a) If the differential equation

$$\omega''(z)+p(z)\omega'(z)+q(z)\omega(z)=0$$

has two linearly independent regular solutions $\omega_1(z)$ and $\omega_2(z)$ in a nbd of $z=0$ (an isolated singularity) then prove that $(z-z_0)p(z)$ and $(z-z_0)q(z)$ both are analytic at $z=z_0$.

b) Define self-adjoint operator. Show that, $\frac{d^2}{dx^2}$ is a self-adjoint operator. 5+3

3. a) Define dirac-delta function $f(x)$. Show that, $\int_{-\infty}^{+\infty} f(x)\delta(x)dx=f(0)$ where, $f(x)$ is a continuous and bounded function.

b) Show that, $f(t,x)=(x+x^2)\frac{\cos t}{t^2}$, satisfies Lipschitz's condition in $R:|x|\le1,\ |t-1|<\frac{1}{2}$.

5+3

4. a) Using Green's function method, solve the following differential equation:

$$y^{IV}(x)=1$$

Subject to boundary conditions

$$y(0)=y'(0)=y''(0)=y'''(1)=0.$$

b) Prove that, Green's function is symmetric.

5+3

5. a) Prove that

$$\int_{-1}^{1} P_m(z) P_n(z)\,dz = \frac{2}{2n+1}\delta_{mn}$$

Where δ_{mn} and $P_n(z)$ are the Kroneker delta and Legendre's polynomial respectively.

b) Find the general solution of the PDE

$$(x-y)p+(y-x-z)q=z.$$ 5+3

6. a) Determine the surface which passes through $4z+x^2=0$, $y=0$ and an integral solution of the expression $z=p^2-q^2$.

b) Solve $D(D-2D')(D+D')z=e^{x+2y}\left(x^2+4y^2\right)$.

5+3

7. a) A thin homogenous and perfectly flexible string stretched with fixed end points $x=0$ and $x=L$ with initial positions $u(x,\,0)=10\sin\dfrac{\pi x}{L}$ and then released. Find the displacement of the string at any point at any time.

b) State and prove Maxima-Minima principle of a harmonic functions. 5+3

8. a) Find the solution of Laplace equation $\nabla^2\psi=0$ in a plane of polar co-ordinates.

b) Reduce the polar form of the parabolic equation from its cartesian form

$$\frac{\partial T}{\partial t}=K\frac{\partial^2 T}{\partial x^2} \text{ in } -\infty<x<\infty,\ t>0.$$ 5+3

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