P.G. 1st Semester - 2016

MATHEMATICS

(CBCS)

Paper : MMATCCT-104

Full Marks : 40 Time : 2 Hours

The figures in the right-hand margin indicate marks.
Candidates are required to give their answers in their own words as far as practicable.
Notations and Symbols have their usual meaning.

Answer any **five** questions. 8×5=40

1. a) Find two linearly independent integrals of the differential equation

$$z^2w'' + zw' + (z^2-1)w = 0$$

in a neighbourhood of the origin.

b) Show that all the roots of $P_n(x)=0$ are real and lie between –1 and +1. 5+3

2. a) Show that the point of infinity is a regular singular point of the hypergeometric differential equation

$$z(1-z)\frac{d^2w}{dz^2} + \{c-(a+b+1)z\}\frac{dw}{dz} - abw = 0$$

and obtain a regular integral of the above equation about $z \to \infty$.

b) Prove that,

$$\frac{d}{dz}F(a,b,c,z) = \frac{ab}{c}F(a+1,b+1,c+1,z).$$

1+4+3

3. a) For a Legendre polynomial $P_n(z)$ prove that

$$P_n(z) = \frac{1}{2^n \lfloor n}\frac{d^n}{dz^n}(z^2-1)^n$$

b) Prove that the Bessel functions of the first kind $J_n(x)$ satisfies the relation

$$J_n(x) = \frac{1}{\pi}\int_0^{\pi}\cos(n\theta - x\sin\theta)d\theta.$$ 4+4

4. a) Define adjoint and self adjoint operator for a second order ordinary differential equation. Show that $\frac{d^2}{dx^2}$ is a self adjoint operator.

b) Show that if u and v are solutions of the self adjoint differential equation $(py')' + qy = 0$, then $p(uv' - u'v) =$ constant. (2+2)+4

5. a) Reduce the partial differential equation $x^2 z_{xx} + 2xyz_{xy} + y^2 z_{yy} = 0$ to its canonical form.

b) Using the transformations $u = \log x$, $v = \log y$, reduce the equation $\left(x^2 D^2 + 2xyDD' + y^2 D'^2\right) z = xy^2$ to a partial differential equation with constant coefficients and hence solve it. 4+4

6. a) Find the potential $\Psi(x, y, z)$ in a rectangular box defined by $0 \le x \le a,\ 0 \le y \le b,\ 0 \le z \le c$ if the potential is zero in all sides and the bottom, while $\Psi = f(x, y)$ on the top $z = c$ of the box.

b) What do you mean by special mean? 6+2

7. a) In a one dimensional infinite solid $-\infty < x < \infty$, the surface $a < x < b$ is initially at temperature T_0 and at zero temperature outside the surface. Show that the expression of temperature is given by

$$T(x,t) = \frac{T_0}{2}\left[\operatorname{erf}\left(\frac{b-x}{\sqrt{4kt}}\right) - \operatorname{erf}\left(\frac{a-x}{\sqrt{4kt}}\right)\right]$$

where, 'erf' is the error function and 'k' is the conductivity of the solid.

b) Reduce the Laplace equation $\nabla^2 u = 0$ in the plane of polar coordinates (r, θ). 4+4

8. a) Obtain the D'Alembert's solution of the one dimensional wave equation

$$\frac{\partial^2 u}{\partial x^2} = c^2 \frac{\partial^2 u}{\partial t^2},\ -\infty < x < \infty,\ t \ge 0$$

subject to the initial condition

$$u(x,0) = \eta(x) \text{ and } \frac{\partial u}{\partial t}(x,0) = v(x).$$

b) Define 'Green's function'. Show that Green's function $G(r, r^1)$ has symmetric properties. 4+4
