

**P.G. 1st Semester - 2016****PHYSICS****(CBCS)****Paper : 103****Quantum Mechanics-I**

Full Marks : 40

Time : 2 Hours

*The figures in the right-hand margin indicate marks.**Candidates are required to give their answers in their own words as far as practicable.*Answer Q. No. 1 and any **three** from the rest.

1. Answer any **five** questions: 2×5=10
- a) Consider two Hermitian operators A & B. Then show the condition when AB will be Hermitian.
- b) If  $\vec{A}$  and  $\vec{B}$  are two vector operators whose components commute with  $\sigma_i$ ; then prove that
- $$(\vec{\sigma} \cdot \vec{A})(\vec{\sigma} \cdot \vec{B}) = \vec{A} \cdot \vec{B} + i\vec{\sigma} \cdot (\vec{A} \times \vec{B})$$
- $[\sigma_i (i = x, y, z)]$  denote the Pauli spin matrices
- c) Find the matrix representation values of  $J_+$  &  $J_-$  for  $J = 2$ .
- d) What is C. S. C. O?

- e) Find  $[x, P_x^2]$  where x and  $p_x$  represent position and momentum operators.
- f) If H is the Hamiltonian operator of a particle, then it commutes with the time evolution operator.
- g) Show that a coherent state is a eigen state of annihilation operator 'a'.
- h) Prove that the expectation value of an operator remains same under unitary transformation.

2. a) Show that for a linear harmonic oscillator, the annihilation operator a and the creation operator  $a^+$  are not Hermitian but  $a^+a$  is.

- b) Show that  $a|n\rangle = \sqrt{n}|n-1\rangle$

$$H|n\rangle = \left(n + \frac{1}{2}\right)\hbar\omega|n\rangle$$

- c) Derive the expression of energy of linear harmonic oscillator using operator method.

2+2+6=10

3. a) Find the connection formulae for the W.K.B. solution across a turning point  $x = a$  if  $E > V(x)$  for  $x < a$ .
- b) Obtain by the W.K.B. method, the energy levels in the one dimensional potential

$$V(x) = \begin{cases} \frac{V_0|x|}{a} & \text{for } -a \leq x \leq a \\ = V_0 & \text{for } |x| > a \end{cases}$$

5+5=10*[Turn Over]*

162/Phs.

*[ 2 ]*

4. a) Prove  $J_+ |jm\rangle = \sqrt{(j-m)(j+m+1)} |j, m+1\rangle$ .  
 b) Compute from the first principle, obtain the Clebsch-Gordan co-efficient for  $j = \frac{3}{2}$ ,  
 where  $j_1 = 1$  and  $j_2 = \frac{1}{2}$  for all possible in values.  
 c) Find  $[J^2, J_z]$ . 2+6+2=10
5. A particle lies in a delta function potential well. Obtain the expression of the quantized energy level. Also derive the expression of wavefunction of the particle within and outside the well. Hence normalize the wave function. 5+3+2=10
6. a) Establish Heisenberg equation of motion. Write the difference between the Schrödinger and Heisenberg pictures of equation of motion.  
 b) An one dimensional oscillator is perturbed by its anharmonicity of  $bx^4$ , where  $b$  is a constant and  $x$  represents its position. Obtain the ground state energy correction of the oscillator. (5+2)+3=10

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