## U.G. 3rd Semester Examination - 2019

## **MATHEMATICS**

## **|HONOURS|**

Course Code: MATH(H)CC-05-T

Full Marks: 60

Time:  $2\frac{1}{2}$  Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Symbols and notations have their usual meanings.

Answer any ten questions:

 $2 \times 10 = 20$ 

1) Use sequential criterion for limits to show that the following limit does not exist

$$\lim_{x\to 0} \frac{1}{x} \sin \frac{1}{x}.$$

- Give example of function f and g which are not continous at a point c∈ R but the sum ftg is continous as c.
- jii) Using ε-δ definition, show that

$$\lim_{x \to \infty} \frac{[x]}{x} = 1.$$

[Turn over]

- (iv) Verify what her (IR, d) is a metric space, where  $d(x, y) = [x^2 y^2]$ ,  $\forall x, y \in IR$ .
- Define diameter of a set in a metric space (x, d).
- vi) Does f'(c) = 0 always imply existence of an extremum of f at c? Justify.
- Give an example of a function which has a jump discontinuity in its domain of definition.
- Niii) Show that the equation f(x)=xe<sup>1</sup>-2 has a root in [0, 1].
  - ix) Expand log sin (x+h) in power of h by Taylor's Theorem.
- (Xx) Give geometrical interpretation of Lagrange's Mean Value Theorem.
- - xii) For a metric space X, show that a point  $a \in X$  is a cluster point of  $A \subset X$  if there exists  $\{a_n\}_{n=1}^{\infty}$  in A such that  $\lim_{n \to \infty} a_n = a$ .

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(2)

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- xifi) Show that there does not exist a function  $\phi$  such that  $\phi'(x) = f(x)$ , where  $f(x) = x \{x\}$ ,  $x \in [0, 2]$ .
- Discuss the applicability of Rolle's theorem for  $f(x) = 2 + (x-1)^{\frac{1}{2}}$  in [0, 2].
- xv) Show that f(x)=|x+2| is continuous at x=-2 but not differentiable at this point.
- 2. Answer any four questions:

5 - 4 = 20

I.et f be a continuous function on [a, b] and c be any real number between f(a) and f(b), then show that there exist a real number x in (a, b) such that f(x)=c.

Construct an example to show that continuity of f is not necessary for the existence of such x as above.

3+2

- in State and prove Rolle's theorem.
- iii) Find the maxima and minima of the function  $f(x) = \sin x + \frac{1}{2}\sin 2x + \frac{1}{3}\sin 3x \text{ for all } x \in [0, \pi].$

Define a Metric space show that  $(\mathbb{R}^2, d)$  is a metric space, where the metric  $d: \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}$  is defined by  $d(x, y) = |x_1 - y_1| + |x_2 - y_2|$ ;  $x,y \in \mathbb{R}^2$  when  $x = (x_1, x_2), y = (y_1, y_2)$ .

Suppose n-th derivative of a function f exists finitely in a closed interval [a, a+h]. Then show that there exists a positive proper fraction θ satisfying the relation

$$f(a+h) = f(a) + hf'(a) + \frac{h^2}{2!}f''(a) + ...$$

$$+ \frac{h^{n-1}}{(n-1)!}f^{n-1}(a) + \frac{h^n}{n!}f''(a+\theta h).$$

vi) Let f: [a,b] → IR be such that f has a local extremum as an interior point c of [a,b]. If f'(c) exists, then prove that f'(c)=0.

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(3)

[Turn over]

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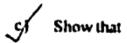
(4)

Answer any two questions from Question No. 3 to Question No. 6: 10×2=20

- A function f: [0, 1] → [0, 1] is continous on [0,1].
   Prove that there exists a point c in [0,1] such that f(c)=c.
  - b) Show that every uniformly continous function on an interval is continous on that interval, but the converse is not true.
  - Prove that the function  $f(x) = \frac{1}{x}$ ,  $x \in (0, 1]$  is not uniformly continous on (0, 1]. 3+(2+2)+3

f. a) If f' and g' exist for all 
$$x \in [a, b]$$
 and  $g'(x) \neq 0 \ \forall \ x \in (a, b)$ , then prove that for some  $c \in (a, b)$ ,  $\frac{f(c) - f(a)}{g(b) - g(c)} = \frac{f'(c)}{g'(c)}$ .

b) Obtain the Maclaurin's series expansion of log(1+x), -1 < x ≤ 1.</p>



$$x - \frac{x^2}{2} < \log(1+x) < x - \frac{x^2}{2(1+x)}, x > 0.$$

- a) Prove that in a metric space every open ball is an open set and every closed ball is a closed set.
  - b) Define the following with example:
    - i) Subspace of a metric space
    - ii) Separable metric space (3+3)+(2+2)
- 6. a) Show that the function f on [0, 1] defined as  $f(x) = \frac{1}{2^n} \text{ when } \frac{1}{2^{n+1}} < x \le \frac{1}{2^n}, \ n = 0, 1, 2, ...,$   $f(0) = 0 \text{ is discontinuous at } \frac{1}{2}, \left(\frac{1}{2}\right)^2, \left(\frac{1}{2}\right)^2, ....$ 
  - b) Show that  $\lim_{a\to\infty} a^a \cdot \sin \frac{b}{a^a} = \begin{cases} 0 & \text{if } 0 < a < 1 \\ b & \text{if } a > 1 \end{cases}$ .

Prove that in a metric space (X, d), the interior of a set A ⊂ X is the largest open subset of A.
 3+3+4

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