UNIVERSITY OF NORTH BENGAL

B.Sc. Honours 2nd Semester Examination, 2020

CC3-MATHEMATICS

REAL ANALYSIS

MATH 21 HCC-III

Full Marks: 60

ASSIGNMENT

The figures in the margin indicate full marks.
All symbols are of usual significance.

Answer Question No. 1 and any *four* from the rest

1. Attain ***all*** the following questions: 2×6 = 12

(a) What is a countable set?

(b) Give an example of a set which do not contain its supremum and infimum.

(c) Find the derived set of an open interval (a, b) of real numbers.

(d) Let $x_n = \dfrac{3n+1}{n+2}$. Prove that the sequence $\{x_n\}$ is a strictly monotone increasing.

(e) Define the greatest lower bound of a set of bounded set of real numbers.

(f) Give an example of a sequence $\{x_n\}$ which does not converge but $\{|x_n|\}$ converges.

2. Prove that the set of natural numbers N is unbounded above. Prove that for any $\epsilon > 0$ there exist a natural number $n \in N$ such that $0 < \frac{1}{n} < \epsilon$. If $x \in$ a bounded set S of real numbers, then prove that the set T formed by the element $-x$ is also bounded and $\sup T = -\inf S$. 3+4+5=12

3. Show that if B is a countable subset of an uncountable set A then $A - B$ is uncountable. Show that derive set is closed. Prove that the union of two countable sets is countable. 3+4+5=12

4. Prove that every close interval $[a, b]$ is a closed set. Find the derive set of $S = \left\{\frac{1}{2^m} + \frac{1}{3^n} : m, n \in N\right\}$. Let S be a closed subset of R, then prove that $\bar{S} = S \cup S'$, S' being the derived set of S. 3+4+5=12

5. Use Cauchy's criterion to prove that the sequence $\{x_n\}$, $x_n = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots\ldots + (-1)^n \frac{1}{n}$ is convergent. Show that every convergent sequence is bounded. Show that every compact set of real numbers is closed and bounded. 3+3+6=12

6. Prove that $\lim_{n\to\infty} \frac{1}{n}(n!)^{1/n} = \frac{1}{e}$. Show that every monotonically increasing bounded sequence is convergent. Prove that the sequence $\{x_n\}$ defined by $x_1 = \sqrt{6}$, $x_{n+1} = \sqrt{6 + x_n}$ for $n \geq 1$ converges to 3. 3+3+6=12

7. Let S be a non-empty subset of R having a limit point ξ. Show that there exists a sequence $\{x_n\}$ of distinct elements of S such that $\lim x_n = l$. Test for convergence the following series 6+6=12

$$1 + \frac{(1!)^2}{2!}x + \frac{(2!)^2}{4!}x^2 + \frac{(3!)^2}{6!}x^3 + \ldots\ldots\ldots$$

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