P.G. 1st Semester - 2018

MATHEMATICS

Paper : MMATCCT101

Full Marks : 40 Time : 2 Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meaning.

Answer any **five** questions. $8\times5=40$

1. a) Prove that if G is a finite abelian group and p is a prime divisor of O(G), then there exists atleast one element $a \in G$ of order p.

   b) Show that the groups (Q, +) (Q^+, **.**) are not isomorphic, where Q is the set of all rational numbers and Q^+ is the set of all positive rational numbers. 5+3=8

2. a) Let A and B be two cyclic groups of order m and n respectively. Then show that $A\times B$ is cyclic if and only if gcd(m, n)=1.

   b) Find all abelian groups of order 3^4.

   c) Find the derived subgroup of permutation group S_3. 4+2+2=8

3. a) Show that every group of order p^2q is solvable where p and q are distinct primes.

   b) Examine whether S_4 is a solvable group or not. 5+3=8

4. a) Define inner automorphism of a group G.

   b) Prove that any group of order 15 is cyclic.

   c) Show that every finite p-group is nilpotent. 2+3+3=8

5. a) Show that any two elements a, b in an Euclidean domain R have gcd. Hence deduce that a and b are relatively prime iff $\lambda a+\mu b=1$ for some $\lambda, \mu \in R$.

   b) Find associates of $1+2\sqrt{-5}$ in ring $\mathbb{Z}\left[\sqrt{-5}\right]$. 5+3=8

6. Prove that every principal ideal domain is unique factorisation domain but the converse is not true in general. 8

7. a) If K/F and L/K are two field extensions, then show that [K : F] and [L : K] are finite iff [L:F] is finite. Moreover show that

$$[L:F]=[L:K][K:F]$$

b) Show that a finite extension of prime degree is a simple extension. 5+3=8

8. a) Normal extension of a normal extension of a field is also normal extension.– Justify your answer.

b) If F is a field of prime characteristic, then show that F is perfect iff every element of F has p-th root in F. 3+5=8