P.G. 1st Semester - 2018

MATHEMATICS

Paper : MMATCCT102

Full Marks : 40 Time : 2 Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meaning.

Answer any **five** questions: 8×5=40

1. a) Let f be a bounded variation of [a, b] and $c \in (a, b)$. Then prove that

   i) f is of bounded variation on [a, c] and [c, b];

   ii) $V_f(a, b)=V_f(a, c)+V_f(c, b)$

   b) Let $f:[0, 3] \to \mathbb{R}$ be defined by $f(x)=x^2-4x+3$, $x \in [0, 3]$. Show that f is a function of bounded variation on [0, 3]. Also calculate $V_f[0, 3]$.

   5+3

2. a) Let f be a function on [a, b] and α is monotone increasing function on [a, b]. Then show that, f is integrable with respect to α on [a, b] iff for every $\varepsilon > 0$ $\exists$ a partition P of [a, b] such that

$$U(p, f, \alpha) - L(p, f, \alpha) < \varepsilon .$$

   b) Evaluate, $\int_0^4 (x^2 + x + 1) d[x]$. 5+3

3. a) Assume $f \in R(\alpha)$ on [a, b] and assume that α has a continuous derivative α' on [a, b]. Then prove that Riemann integral $\int_a^b f(x)\alpha'(x)dx$ exists and $\int_a^b f(x)d\alpha(x) = \int_a^b f(x)\alpha'(x)dx$.

   b) Define Rectifiable curves.

   c) Evaluate $\int_1^4 (x - [x])dx^2$. 4+2+2

4. a) If directional derivative exists for a function f then all the partial derivative exists for f.– Give justification of the above statement.

   b) Show that the converse of the above statement is not true in general.

   c) Give an example of a function which have finite directional derivative for every direction but fail to be continuous at some point.

d) Write down the relation between total derivate and directional derivative for the direction $\vec{u}$.

2+2+2+2

5\. a) Let u and v be two real-valued functions defined on a subset S of the complex plane. Assume also that u and v are differentiable at $c \in \text{int}(s)$ and that the partial derivatives satisfy the Cauchy Riemann equations at C. Then show that the function f=u+iv has a derivative at c and $f'(c) = D_1u(c) + i\,D_1V(c)$.

b) Let $T:\mathbb{R}^n \to \mathbb{R}^m$ be a linear function. Then express T in matrix form. 5+3

6\. Let f=(f_1, f_2, f_3) be the vector values function defined as follows:

$$f_k(x_1, x_2, x_3) = \frac{x_k}{1 + x_1 + x_2 + x_3} \quad \begin{matrix}(k = 1,2,3), x_i \in R\forall i \\ (x_1 + x_2 + x_3 \neq -1)\end{matrix}$$

a) Show that $J_f(x_1, x_2, x_3) = (1 + x_1 + x_2 + x_3)^{-4}$.

b) Show that f is one to one.

c) Compute f^{-1} explicitly. 2+3+3

7\. State and prove the sufficient condition for existence of extreme value of a function with two variables.

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8\. a) Show that the points, on the ellipse $5x^2 - 6xy + 5y^2 = 4$ which the tangents are at the greatest distance from the origin, are (1, 1) and (–1, –1).

b) If $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$, show that a stationary point of $a^3x^2+b^3y^2+c^3z^2$ is given by ax=by=cz and this is an extreme point if abc(a+b+c) is positive.

4+4

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