P.G. 1st Semester - 2018

MATHEMATICS

Paper : MMATCCT104

Full Marks : 40 Time : 2 Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meaning.

Answer any **five** questions: 8×5=40

1. a) Discuss the singularities of the equation

$$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+\left(x^2-n^2\right)y=0 \text{ at } x=\infty.$$

b) By Frabenius method, find the series solution of $9x(1-x)y''-12y'+4y=0$ about origin.

2+6

2. a) Prove that, Legendre polynomial $P_n(x)$ can be written as $P_n(x)=\frac{1}{2^n\underline{|n}}\frac{d^n}{dx^n}\left(x^2-1\right)^n$.

b) Prove the recurrence relation

$$\left(1-x^2\right)P_n'(x)=(n+1)\left(xP_n(x)-P_{n+1}(x)\right).$$

4+4

3. a) Show that generating function of the Bessel polynomial $J_n(x)$ is given by $\exp\left\{\frac{1}{2}x\left(z-\frac{1}{z}\right)\right\}$.

b) Prove that $J_0(x)=\frac{1}{\pi}\int_0^{\pi}\cos(x\sin\theta)\,d\theta$.

4+4

4. a) Prove that, for the Hermite polynomial $H_n(x)$,

$$\int_{-\infty}^{\infty}e^{-x^2}H_n(x)H_m(x)\,dx=\begin{cases}0, & m\neq n\\ \sqrt{\pi}\,2^n\underline{|n}, & m=n.\end{cases}$$

b) Define Green function in polar co-ordinate system. Hence or otherwise show that,

$$\nabla^2\left(\frac{1}{r}\right)=-4\pi\delta(r),$$

where $\delta(r)$ is the dirac-delta function. 4+4

5. a) Find the complete integral of the p.d.e. $p^2q^2+x^2y^2=x^2q^2\left(x^2+y^2\right)$.

b) Solve the p.d.e.

$$\left(D^2+DD'-6D'^2\right)z=x^2\sin(x+y).$$

4+4

6. Reduce the following p.d.e. to its canonical form. Hence solve it.

$$\left(1+x^2\right)Z_{xx}+\left(1+y^2\right)Z_{yy}+xZ_x+yZ_y=0$$ 5+3

7. a) A thin rectangular homogeneous thermally conducted plate occupies the region $0 \le x \le a$, $0 \le y \le b$, The edge y=0 held at temperature Tx(x–a), where T is constant and other edges are maintained at 0^{o}. Find the steady state temperature inside the plate.

   b) Reduce the Laplace equation $\nabla^2 u = 0$ in the plane of polar co-ordinates (r, θ). 4+4

8. a) Find the D-Alembert's solution of the initial value wave equation $\frac{\partial^2 u}{\partial x^2} = c^2 \frac{\partial^2 u}{\partial t^2}$, $-\infty < x < \infty$, $t \ge 0$ subject to the initial conditions

$$u(x, 0) = \eta(x) \text{ and } \frac{\partial u}{\partial t}(x, 0) = \gamma(x).$$

   b) Prove the uniqueness of the solution of the wave equation. 4+4

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