P.G. 1st Semester - 2017

MATHEMATICS

Paper : MMATCCT-106

Full Marks : 40 Time : 2 Hours

The figures in the right-hand margin indicate marks. Candidates are required to give their answers in their own words as far as practicable.

Answer any **five** questions. 8×5=40

1. a) Every closed subspace of a compact space is compact.

   b) Let $\{A\alpha\}_{\alpha\in\wedge}$ be a collection of compact subsets of a Housdroff space (x,τ). Prove that $\bigcap_{\alpha\in\wedge} A\alpha$ is compact.

   c) Let $\{x_n\}$ be sequence in a topological space X and $x_n \to x \in x$. Show that $\{x_n : n\in\mathbb{N}\}\cup\{x\}$ is compact subset of X.

   3+3+2

2. a) Show that every finite set in a Hausdroff space is closed.

   b) Show that a topological space X is Hausdroff if and only it the diagonal $\Delta=\{(x,x) : x\in X\}$ is closed in X×X. 3+5=8

3. a) Suppose that X and Y are two topological spaces and $f: X\to Y$ is a continuous function. If x is a limit point of a subset A of X, then f(x) is a limit point of f(A) – prove or disprove.

   b) Suppose that X be a topological space and A, B are closed subsets in X such that $X = A\bigcup B$. Y be another topological space, $f: A\to Y$ and $g \cdot B\to Y$ be continuous function. If $f(x)=g(x)\forall x\in A\cap B$ then show that the function $h: X\to Y$ defined by

   $$h(x)=\begin{cases} f(x) & \text{if } x\in A \\ g(x) & \text{if } x\in B \end{cases}$$

   is a continuous function. 4+4=8

4. a) Show that composition of two homeomorphisms is a homeomorphism. Is $\mathbb{R}^2$ homeomorphic to S^2?— Justify.

   b) Let $\mathbb{R}^\omega$ the countably infinite product of $\mathbb{R}$ with boy topology. Where $\mathbb{R}$ is equipped with standard topology. Then prove or disprove that the function

$f : \mathbb{R} \to \mathbb{R}^{\omega}$ defined by f(t)=(t, t, t,...)

is continuous. 4+4=8

5. a) Let X be a topological space satisfying the first countability axion. A be subset of X, $x \in X$. Show that there is a sequence of points of A converging to x if and only if $x \in \bar{A}$.

   b) What do you mean by second countable space? 6+2=8

6. a) Assume $\mathbb{R}$ is equipped with standard topology and Q is subspace of $\mathbb{R}$. Then show that $\mathbb{R}^2 - Q^2$ is connected.

   b) Prove or Disprove $\mathbb{R}$ with lower limit topology is connected. 5+3=8

7. a) Define component of a topological space X.

   b) Show that a topological space X is locally connected if and only if for every open set U in X, each component of U is open in X.

   2+6=8

8. a) Let $\mathbb{R}$ is equipped with standard topology. Then show that $\left\{\frac{1}{n} : n \in \mathbb{N}\right\} \vee \{0\}$ is compact in $\mathbb{R}$ but $\left\{\frac{1}{n} : n \in \mathbb{N}\right\}$ is not compact in $\mathbb{R}$.

   b) Show that every cofinite topological space is compact. 5+3=8

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