# P.G. 1st Semester-2018

## PHYSICS

### Paper : MPHYCCT103

Full Marks : 40 Time : 2 Hours

*The figures in the right-hand margin indicate marks.*

*Candidates are required to give their answers in their own words as far as practicable.*

**Answer Q. No. 1 and any three from the rest.**

1. Answer any **five** from the following: 2×5=10

a) If $\sigma_x, \sigma_y, \sigma_z$ are the Pauli spin matrices, show that $\sigma_x \sigma_y \sigma_z = i$.

b) Show that $\hat{J}_+ \hat{J}_- = \hat{J}^2 - \hat{J}_z^2 + \hbar J_z$

$\hat{J}_+$ and $\hat{J}_-$ are the ladder operators constructed from the angular momentum operators $\hat{J}_x$, $\hat{J}_y$ and $\hat{J}_z$.

c) Formulate Schrödinger's equation of motion in the interaction picture.

d) Prove that $\sigma_i \sigma_j + \sigma_j \sigma_i = 0$ if $i \neq j$.



e) A one-dimensional simple harmonic oscillator is in the state $\psi = \frac{\sqrt{2}}{3}\psi_1 + \frac{2}{3}\psi_3 + \frac{1}{\sqrt{3}}\psi_5$.

If a measurement of energy is done then find the most probable value of energy.

f) Find the matrix representation values $J_+$ and $J_-$ for j=2.

g) Wave function of a particle is given to be

$$\psi(x) = \frac{1}{\sqrt{2a}} \text{ for } |x| < a$$

and $\psi(x) = 0$ for $|x| > a$

Find the uncertainty in position.

h) Find out the explicit expression for the operator $\left(\frac{d}{dx} + x\right)^2$.

2. a) What is the method of variation?

b) For a harmonic oscillator, using a Gaussian wave function $\left(Ae^{-\alpha x^2}\right)$ as a trial wavefunction, estimate the ground state energy.

c) Draw the variation of Energy with respect to $\alpha$. 2+6+2=10

3. a) Find the connection formulae for the W.K.B solution across a turning point x=a if $E > V(x)$ for $x < a$.

b) Obtain by the W.K.B method, the energy levels in the one-dimensional potential

$$V(x) = \frac{V_0|x|}{a} \text{ for } -a \le x \le a$$
$$= V_0 \text{ for } |x| > a$$

5+5=10

4. The Hamiltonian of a linear harmonic Oscillator is $H = \frac{P^2}{2m} + \frac{1}{2}m\omega^2 x^2$ if $a = \frac{1}{\sqrt{2}}(\hat{x} + i\hat{p})$ in atomic unit.

a) Evaluate the commutator $[a, a^+]$

b) Show that H can be put in the form

$$H = \left(a^+ a + \frac{1}{2}\right)\hbar\omega$$

Hence prove $H\langle n\rangle = \left(n + \frac{1}{2}\right)\hbar\omega|n\rangle$.

c) Show that a, $a^+$ are not Hermitian but $a^+a$ is.

d) Show that a coherent state is a eigen state of annihilation opeartor a.

e) Write down the Schrödinger equation for Helium atom. 2+3+2+2+1=10

5. a) Obtain the Clebsch-Gordan co-efficients for J=1 and J=0 where $j_1 = 1$ and $j_2 = \frac{1}{2}$ for all possible values from the first principles.



b) Deduce the experession such as we can say that total angular momentum $\vec{J}$ of a system is the generator of infinitesimal rotations. 6+4=10

6. a) Establish Heisenberg equation of motion. Write the difference between the Schrödinger and Heisenberg pictures of equation of motion.

b) An one-dimensional oscillator is perturbed by its anharmonicity of $bx^4$, where b is a constant and x represent its position. Obtain the ground state energy correction of the oscillator.

(4+2)+4=10

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