2018
BCA
[HONOURS]
(Mathematics)
Paper : BCA-102

Full Marks : 80 Time : 4 Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

1. Answer any **ten** question: $2\times 10=20$

a) Let $f: R \to R$ be defined by $f(x)=2x+1, x\in R$. Find $f^{-1}\left(\frac{1}{2}\right)$.

b) Find the value of $\begin{vmatrix} 1 & 2 & 3 \\ 2 & 4 & 7 \\ 3 & 6 & 9 \end{vmatrix}$

c) Find the intersection of the following sets

$A=\{x : x \text{ is a prime integer} \le 11\}$
$B=\{x : x \text{ is an odd integer} < 12\}$

d) Find the nature of the conic

$$r=\frac{8}{4-5\cos\theta}$$

e) Find the straight line passing through origin and perpendicular to $x+y=1$.

f) Find the straight line parallel to the line $3x+4y=12$ and passing through (2, 3).

g) Find the equation whose roots are 1, 2, 3, 4.

h) Find the nature of the roots of $x^3+x^2-5x-1=0$.

i) Find the exponential form $\frac{1}{2}+i\frac{\sqrt{3}}{2}$.

j) If $\bar{A}=3\hat{i}-4\hat{j}+5\hat{k}$ and $\bar{B}=\hat{i}+2\hat{j}+\hat{k}$ find $\bar{A}\cdot\bar{B}$.

k) Find the value of k so that the equation $6x^2+xy+ky^2+2x-31y-20=0$ represents a pair of straight lines.

l) Find A+B and A.B if possible, when

$$A=\begin{bmatrix} 1 & 2 & 5 \\ -1 & 3 & -4 \end{bmatrix} \text{ and } B=\begin{bmatrix} 3 & -2 & 1 \\ 0 & -1 & 4 \\ 5 & 2 & -1 \end{bmatrix}$$

m) Define cross product between two vectors.

n) Solve the equation $x^3-3x^2+4=0$, two of the roots being equal.

o) Define subspace.

2. Answer any **four** questions: 5×4=20

a) Let A, B, C be subsets of a universal set S then prove that

$$A\times(B\cap C)=(A\times B)\cap(A\times C)$$

b) If $a,b\in(G,*)$, then prove that $(a*b)^{-1}=b^{-1}*a^{-1}$

c) Show that the map $f:R\to R$ defined by $f(x)=3x+5,\ x\in R$ is bijective (R is the set of reals)

d) Prove that the set $S=\{1,\omega,\omega^2\}$ where $\omega^3=1$ forms an Abelian group with respect to multiplication.

e) Prove that intersection of two subgroup is a subgroup.

f) Find the general values of $(1+i)^i$.

3. Answer any **four** questions: 5×4=20

a) Determine the matrices A and B

where $A+2B=\begin{bmatrix}1 & 2 & 0\\ 6 & -3 & 3\\ -5 & 3 & 1\end{bmatrix}$

and $2A-B=\begin{bmatrix}2 & -1 & 5\\ 2 & -1 & 6\\ 0 & 1 & 2\end{bmatrix}$

b) Solve by Cramer's rule

$$\begin{aligned}x+y-z&=6\\ 2x-3y+z&=-1\\ 3x-4y+2z&=-1\end{aligned}$$

c) Find the condition that roots of the equation $x^3-px^2+qx-r=0$ will be in Geometric Progression (G.P).

d) Solve $x^3-6x-9=0$ by Cardan's method.

e) If α be a root of the cubic $x^3-3x+1=0$ then show that the other roots are — (α^2-2), and $(2-\alpha-\alpha^2)$.

f) Show that

$$\begin{vmatrix}1+a_1 & 1 & 1\\ 1 & 1+a_2 & 1\\ 1 & 1 & 1+a_3\end{vmatrix}=a_1a_2a_3\left(1+\frac{1}{a_1}+\frac{1}{a_2}+\frac{1}{a_3}\right)$$

g) Find the unit vector which is perpendicular to each of the vectors

$$\left(2\hat{i}-\hat{j}+\hat{k}\right)\text{and}\left(3\hat{i}+4\hat{j}-\hat{k}\right).$$

4. Answer any **two** questions: $10\times 2=20$

a) Reduce the equation $14x^2-4xy+11y^2-44x-50y+75=0$ to canonical form and determine the nature of the conic. 10

b) What does the equation $11x^2+16xy-y^2=0$ become on turning the axes through an angle $\tan^{-1}\frac{1}{2}$? 10

c) i) Find the angle between the vector $\bar{\alpha}=(-2,1,2)$ and $\bar{\beta}=(-2,-2,1)$.

ii) If $ax^2+2hxy+by^2+2gx+2fy+c=0$ represents two lines equidistance from origin then show that

$$f^4-g^4=c\left(bf^2-ag^2\right).$$ 3+7=10

d) The tangent at two points P and Q of the parabola $\frac{l}{r}=1+\cos\theta$ meet at T show that sp. $SQ=ST^2$ where S is focus. 10