2016
BCA
[HONOURS]
(Mathematics)
Paper : BCA-102

Full Marks : 80 Time : 4 Hours

The figures in the right-hand margin indicate marks. Candidates are required to give their answers in their own words as far as practicable.

1. Answer any **ten** questions: $2\times10=20$

a) If $A=\{1, 2, 3, 4, 5\}$, $B=\{3, 4, 5, 6, 7, 8\}$ then find $A\cap B$ and $A-B$.

b) If the function $f:R\to R$ be defined by $f(x)=x^2-x+1$ then find $f^{-1}(7)$.

c) Define complement of a set.

d) Find the equation whose roots are 1, –2, 3, –4.

e) If w is a root of the equation $x^3-1=0$ then find $\frac{w^5+w^4+w^3+w^2+w+1}{w^2+w+1}$.

f) Find the exponential form of $\frac{1}{\sqrt{2}}+i\frac{1}{\sqrt{2}}$.

g) Find the nature of the root of the equation $x^4+x^3+x^2+1=0$.

h) Show that $\begin{vmatrix} 2 & 0 & 1 \\ 1 & 0 & 3 \\ 0 & 1 & 4 \end{vmatrix}=\begin{vmatrix} 0 & 1 & 2 \\ 3 & 0 & 1 \\ 1 & 4 & 0 \end{vmatrix}$.

i) If possible, then find $A+B$ and AB, where

$$A=\begin{bmatrix} 1 & 2 & 5 \\ -1 & 3 & -4 \end{bmatrix},\ B=\begin{bmatrix} 3 & -2 & 1 \\ 0 & -1 & 4 \\ 5 & 2 & 0 \end{bmatrix}.$$

j) Find the remainder when $x^4+4x^2-9x+21$ is divisible by $2x-7$ (using Synthetic Division).

k) Find the condition that the cubic equation $x^3-px^2+qx-r=0$ should have its roots in G.P.

l) Find the nature of the conic $\frac{8}{r}=4-5\cos\theta$.

m) Find the straight line passing through origin and perpendicular to $2x-9y=11$.

n) Show that the vectors $9i+j-6k$ and $4i-6j+5k$ are perpendicular to each other.

o) Define dot product between two vectors.

2. Answer any **four** questions: $5\times4=20$

a) Let A, B and C be subsets of a universal set S. Prove that $A\times(B\cup C)=(A\times B)\cup(A\times C)$.

b) Prove that intersection of two subgroups is a subgroup.

c) Prove that the set $S=\{1,-1,i,-i\}$ forms an Abelian group with respect to multiplication.

d) Find the general values of $(1+i)^i$.

e) Find a basis and dimension of subspace of ω where $\omega=\left\{(x,y,z)\in R^3, \begin{array}{l} x+2y+z=0 \\ 2x+y+3z=0 \end{array}\right\}$.

f) Let R be a ring. Then prove that following:

i) $X.0=0.X=0$ for all $X\in R$

ii) $(-X).Y=X.(-Y)=-X.Y$ for all $X, Y\in R$.

3. Answer any **four** questions: $5\times4=20$

a) Prove that

$$\begin{vmatrix} bc-a^2 & ca-b^2 & ab-c^2 \\ ca-b^2 & ab-c^2 & bc-a^2 \\ ab-c^2 & bc-a^2 & ca-b^2 \end{vmatrix}=\left(a^3+b^3+c^3-3abc\right)^2.$$

b) Solve $x^3-30x+133=0$ by Cardan's method.

c) Solve by Cramer's rule:

$$2x+y+z=1$$

$$x-y+2z=-1$$

$$3x+2y-z=4$$

d) Determine the matrices A and B where

$$A+2B=\begin{bmatrix} 1 & 2 & 0 \\ 6 & -3 & 3 \\ -5 & 3 & 1 \end{bmatrix} \text{ and } 2A-B=\begin{bmatrix} 2 & -1 & 5 \\ 2 & -1 & 6 \\ 0 & 1 & 2 \end{bmatrix}.$$

e) If one of the roots of the equation $x^3+px^2+qx+r=0$ equals the sum of the other two then show that $p^3+8r=4pq$.

f) If α be a root of the equation $x^3+3x^2-6x+1=0$ then show that the other roots are $\frac{1}{1-\alpha}, \frac{\alpha-1}{\alpha}$.

4. Answer any **two** questions: $10\times2=20$

a) i) In any conic, prove that the sum of the reciprocals of two perpendicular focal chords is constant.

ii) Show that the lines $r\cos(\theta-\alpha)=p$ and $r\cos(\theta-\beta)=p$ intersect at

$$\left\{p\sec\frac{1}{2}(\alpha-\beta), \frac{\alpha+\beta}{2}\right\}.$$ $5+5=10$

b) i) If $ax^2+2hxy+by^2+2gx+2fy+c=0$ represents two lines equidistance from origin then show that

$$f^4-g^4=c\left(bf^2-ag^2\right).$$

ii) If pair of lines $x^2-2pxy-y^2=0$ and $x^2-2qxy-y^2=0$ be such that each pair bisects the angles between the other pair, prove that $pq+1=0$. $7+3=10$

c) i) Reduce the equation

$$4x^2-4xy+y^2+2x-26y+9=0$$

to canonical form and determine the nature of the conic.

ii) Find the transformed equation of the straight line $\frac{x}{a}+\frac{y}{b}=2$ when the origin is transferred to the point (a, b).

$8+2=10$

d) i) If A and B are two vectors such that $A=3\hat{i}+\hat{j}+2\hat{k}$ and $B=2\hat{i}-2\hat{j}+4\hat{k}$ then find the unit vector perpendicular to each of them; also find the angle between them.

ii) Find a vector X, where X is collinear with the vector $Y=(2, 1, 3)$ satisfying $X.Y=16$. $6+4=10$
